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OSCILLATIONS OF A PLASMA WITH AN ELECTRON BEAM IN AN EXTERNAL ELECTRIC FIELD

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ABSTRACT

Investigation of the problem of the penetration of an external longitudinal electric field into a semibounded plasma containing an electron beam in the absence of instabilities in the system. It is shown that the system of linear equations for the oscillations of this system can be reduced to an integral equation for the electric field and that a solution can be obtained for this equation in the form of a sum of two components. Formulas are obtained for the distribution of the electric field and of small perturbations of the beam velocity and density. The field distribution in the presence of resonance is ascertained.

A great many studies (a detailed bibliography is given, for example, /3 * in the summary of [Ref. 1]) have been devoted to the interaction of charged particle beams with a plasma. An analysis of the dispersion equation showed that the increasing longitudinal waves (Ref. 2-4) are not excited in the case of a rather slow monoenergetic, low-density electron beam in a system consisting of such a beam and a plasma.

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* Note: Numbers in the margin indicate pagination in the original foreign text.

The penetration of an external longitudinal electric field into a semi-bounded plasma with an electron beam when there are no instabilities in the system is investigated (the boundary problem for increasing waves was examined in [Ref. 5]). In a certain sense, this problem represents a generalization of the second portion of the well-known work by L. D. Landau (Ref. 6) to the case of a plasma with a beam. On the other hand, when there is no external electric field this problem can be regarded as a boundary value problem concerning the interaction of a weakly-modulated electron beam with a plasma.

1. Obtaining an Integral Equation. Let the plasma be bounded by a flat wall which ideally reflects particles impacting upon it, and let the electron beam with the charge density ρ_0 and the velocity v_0 with respect to the plasma be propagated perpendicularly to this plane in the depths of the plasma. It is assumed that there is no thermal scatter of the velocities in the beam. Let the x axis lie along the wall in the direction in which the beam is propagated; let u represent the velocity component along this axis.

The distribution function f(u, x) must have the property f(u, 0) = f(-u, 0) at the boundary; we thus employed the distribution function which is integrated over V_y and V_z.

The magnitude of the longitudinal electric field E_1, the perturbation of the density ρ_1, and the velocity v_1 of the beam are also given at the boundary.

If the deviations from equilibrium are small, then the plasma oscillations of the system can be described by the linear equations (Ref. 3)

$$-i\omega f + u\frac{\partial f}{\partial x} - \frac{e}{m}E\frac{df_0}{du} = 0, \quad -i\omega v + v_0\frac{dv}{dx} = -\frac{e}{m}E$$
$$\frac{dE}{dx} = -4\pi e\int_{-\infty}^{\infty} f\,du + 4\pi\rho, \quad -i\omega\rho + \rho_0\frac{dv}{dx} + v_0\frac{d\rho}{dx} = 0 \tag{1.1}$$

Here f is the deviation of the distribution function from the Maxwell distribution function f_0, ρ - the deviation of the beam charge density from the equilibrium value ρ_0, which is assumed to be compensated by the excess positive charge in the plasma, and v - the deviation of the beam velocity from the equilibrium value v_0. The dependence of all the quantities on time is assumed in the form exp $(-i\omega t)$.

The system of equations (1.1) can be reduced to an integral equation with respect to E(x). Each equation in the system must be formally integrated beforehand in order to do this.

Thus, we find from the two latter equations of (1.1): /4

$$v = \left(v_1 - \frac{e}{mv_0}\int_0^x E\chi(-\xi)\,d\xi\right)\chi(x) \qquad \left(\chi(\zeta) = \exp\frac{i\omega\zeta}{v_0}\right) \tag{1.2}$$

$$\rho = \left\{\rho_1 - \frac{\rho_0}{v_0^2}\left[i\omega v_1 x - \frac{e}{m}\int_0^x E\chi(-\xi)\,d\xi - \frac{i\omega e}{mv_0}\int_0^x E\chi(-\xi)(x-\xi)\,d\xi\right]\right\}\chi(x) \tag{1.3}$$

The relationship connecting f and E, which follows from the first equation of (1.1), is not given here. It fully coincides with the equation given by L. D. Landau (Ref. 6), and it is only necessary for obtaining the integral equation. Finally, integration of the second equation of (1.1) yields

$$E = E_1 + \frac{4\pi i}{\omega}\left(e\int_{-\infty}^{\infty} uf(u, x)\,du - \rho_0 v - \rho v_0 + j_1\right) \qquad (j_1 = \rho_0 v_1 + \rho_1 v_0) \tag{1.4}$$

The following relationship [from the first equation of (1.1)] was employed here:

$$i\omega\int_{-\infty}^{\infty} f\,du = \frac{d}{dx}\int_{-\infty}^{\infty} uf\,du$$

Thus, we arrive at the following integral equation:

$$\begin{aligned} E(x) - \int_0^x L(x-\xi)E(\xi)\,d\xi - \int_0^x K(x-\xi)E(\xi)\,d\xi - \\ -\int_x^{\infty} K(\xi - x)E(\xi)\,d\xi + \int_0^{\infty} K(\xi + x)E(\xi)\,d\xi = \psi(x) \end{aligned} \tag{1.5}$$

$$K(\xi) = \frac{4\pi i e^2}{m\omega}\int_0^\infty \frac{df_0}{du}\exp\frac{i\omega\xi}{u}du \quad (\xi>0), \quad L(\xi) = \frac{4\pi e p_0}{m v_0^2}\xi\exp\frac{i\omega\xi}{v_0} \quad (\xi>0)$$

$$\psi(x) = E_1 - \frac{m v_0 v_1}{e}L(x) + \frac{4\pi i j_1}{\omega}\left(1-\exp\frac{i\omega x}{v_0}\right) \tag{1.5 (cont.)}$$

We should note that the function K(ξ) was studied in the work by L. D. Landau (Ref. 6).

2. Integral Representation of the Solution. Let us reduce the integral equation to a form which lends itself to a solution more readily. Thus, the field E(x) can be conveniently represented as the sum of the two terms

$$E(x) = E_\infty + E^\circ(x) \tag{2.1}$$

It can be readily shown that the magnitude of the field for $x \to \infty$ is

$$E_\infty = \frac{E_1 + 4\pi i j_1/\omega}{1-(\omega_-/\omega)^2-(\omega_+/\omega)^2} = \frac{E_1 + 4\pi i j_1/\omega}{\varepsilon} \tag{2.2}$$

where ω_- and ω_+ are the Langmuir plasma frequencies without a beam and an electron beam, respectively, and ε is the dielectric constant of the plasma with a beam. Let us supplementarily formally define the functions K(ξ) and L(ξ) and the unknown function E°(x) in the region of negative argument values:

$$K(-\xi) = K(\xi), \quad L(-\xi) = L(\xi), \quad E^\circ(-x) = -E^\circ(x) \tag{2.3}$$

Then the integral equation for E°(x) can be written in the following form:

$$E^\circ(x) - \int_{-\infty}^{\infty} K(x-\xi)E^\circ(\xi)d\xi \mp \int_0^x L(x-\xi)E^\circ(\xi)d\xi = \pm g(\pm x) \tag{2.4}$$

$$g(x) = \psi(x) - E_\infty + E_\infty\int_0^x [L(\xi)+2K(\xi)]d\xi \tag{2.5}$$

The upper signs here are for the case of x > 0; the lower signs are for /5 the case of x < 0.

Let us solve the integral equation (2.4) by the Fourier method. Multiplying both sides of the equation by exp (-ikx) and integrating over x from $-\infty$ to $+\infty$, we obtain

$$E^{\circ}(k)[1-K(k)]-E_k^{\circ}L_k+E_{-k}^{\circ}L_{-k}=g_k-g_{-k} \tag{2.6}$$

For any value of $\phi(x)$, the symbols $\phi(k)$ and ϕ_k are determined by the equations

$$\begin{aligned} \varphi(k) &= \int_{-\infty}^{\infty} e^{-ikx}\varphi(x)\,dx, \\ \varphi_k &= \int_{0}^{\infty} e^{-ikx}\varphi(x)\,dx \end{aligned} \tag{2.7}$$

It can be readily seen that if $\phi(x)$ is an even function, then its Fourier component is $\phi(k) = \phi_k + \phi_{-k}$. If $\phi(x)$ is an odd function, then $\phi(k) = \phi_k - \phi_{-k}$.

Taking the fact that it is odd into account, we can represent equation (2.6) in the following form

$$E_k^{\circ}[1-L_k-K(k)]-E_{-k}^{\circ}[1-L_{-k}-K(k)]=g_k-g_{-k} \tag{2.8}$$

In order to solve equation (2.8), we must establish a connection between the functions of the argument -k and the complex conjugate functions of the argument k. This is possible if we distinguish between the real and imaginary parts in $E^{\circ}(x)$, $g(x)$, $L(x)$ and $K(x)$, and if we examine the transform corresponding to them separately. Then (2.8) can be reduced to a system of two equations which relate the imaginary parts of certain analytical functions of k. The real parts of these analytical functions can differ only by the constants. However, if one analyzes the behavior of the functions in the case of $|k| \to \infty$, it can be readily shown that the constants equal zero. As a result, we obtain

$$E^{\circ}_k=\frac{g_k}{1-L_k-K(k)}, \qquad E^{\circ}(k)=\frac{g_k}{1-L_k-K(k)}-\frac{g_{-k}}{1-L_{-k}-K(k)} \tag{2.9}$$

Thus, the electric field can be represented in the following form

$$E=E_{\infty}+\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\frac{g_k}{1-L_k-K(k)}-\frac{g_{-k}}{1-L_{-k}-K(k)}\right]e^{ikx}\,dk \tag{2.10}$$

3. Distribution of Electric Field, Small Velocity Perturbations of the Beam and Its Density. Following the procedure given in (Ref. 2,6), let us introduce the functions $K_1(k)$ and $K_2(k)$ which are determined by the relationships

$$K_1(k) = \left(\frac{\omega_-}{\omega}\right)^2 \beta^2 [J_+(\beta) - 1], \qquad K_2(k) = \left(\frac{\omega_-}{\omega}\right)^2 \beta^2 [J_-(\beta) - 1] \tag{3.1}$$

where (Ref. 2)

$$J_+(\beta) = \frac{\beta}{\sqrt{2\pi}} \int\limits_{C_1} \exp\left(\frac{-s^2}{2}\right) \frac{ds}{\beta - s} \qquad \left(\beta = \frac{\omega}{ku^*}, \; u^* = \frac{\sqrt{\theta}}{\sqrt{m}}\right) \tag{3.2}$$

Here θ is the temperature in energy units, m is the electron mass, and the contour C_1 is shown in the figure.

The function $J_-(\beta)$ differs from $J_+(\beta)$ by the fact that the pole is bypassed from above, and not from below, during integration. Therefore, we have

$$J_-(\beta) = J_+(\beta) + i\sqrt{2\pi}\,\beta \exp(-{}^1\!/\!_2\beta^2) \tag{3.3}$$

It can be readily seen that $K(k) = K_1(k)$ in the case of $k > 0$ and $K(k) = K_2(k)$ for $k < 0$. Similarly to this, let us introduce the function /6
$\Pi(k) = K_k - K_{-k}$, and also the functions $\Pi_1(k)$ and $\Pi_2(k)$, which are determined by the following formulas:

$$\Pi_1(k) = \Pi_2(k) - \sqrt{2\pi}\, i \frac{\omega_-^2 \omega}{k^3 u^{*3}} \exp\left(-\frac{\omega^2}{2k^2 u^{*2}}\right)$$
$$\Pi_2(k) = -\frac{\sqrt{2}}{\sqrt{\pi}} \frac{\omega_-^2}{\omega k u^{*2}} + \frac{1}{\sqrt{2\pi}} \frac{\omega_-^2 \omega}{k^3 u^{*3}} \exp\left(-\frac{\omega^2}{2k^2 u^{*2}}\right) \mathrm{Ei}\left(\frac{\omega^2}{2k^2 u^{*2}}\right) \tag{3.4}$$

Here Ei(z) is the integral exponential function.

It can be shown that $\Pi(k) = \Pi_1(k)$ in the case of $k > 0$ and $\Pi(k) = \Pi_2(k)$ for $k < 0$.

Taking (3.1) and (3.4) into consideration, we obtained the following from formula (2.10):

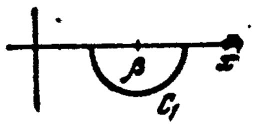

Figure 1

$$E(x) = E_\infty - \int\limits_{-\infty}^{\infty} e^{ikx}\left[\frac{\Phi_+(k)}{1 - L_k - K_2(k)} + \frac{\Phi_-(k)}{1 - L_{-k} - K_2(k)}\right]dk -$$
$$-\int\limits_{0}^{\infty} e^{ikx}\left[K_1(k) - K_2(k)\right]\left\{\frac{F_+(k)}{[1 - L_k - K_1(k)]\,[1 - L_k - K_2(k)]} + \right. \qquad (3.5)$$
$$\left. + \frac{F_-(k)}{[1 - L_{-k} - K_1(k)]\,[1 - L_{-k} - K_2(k)]}\right\}dk$$

Here

$$\Phi_\pm = \frac{iE_\infty\,[L_{\pm k} - \Omega_+^2 + K_2(k) - \Omega_-^2 \pm \Pi_2(k)]}{2\pi k} + \frac{2j_1}{\omega(k \mp \omega/v_0)} \pm \frac{mv_1\omega_+^2}{2\pi e v_0 (k \mp \omega/v_0)^2}$$
$$F_\pm = \pm\frac{iE_\infty}{2\pi k}[1 - L_{\pm k} - K_2(k) + \Pi_2(k) \pm \varepsilon] + \frac{2j_1}{\omega(k \mp \omega/v_0)} \pm \frac{mv_1\omega_+^2}{2\pi e v_0 (k \mp \omega/v_0)^2} \qquad (3.6)$$
$$L_k = \frac{\omega_+^2}{v_0^2 (k - \omega/v_0)^2}, \quad \Omega_+ = \frac{\omega_+}{\omega}, \quad \Omega_- = \frac{\omega_-}{\omega}$$

The complete calculation of the integrals in (3.5) can only be done numerically. However, we can readily obtain the asymptotic formula providing the law by which the field E(x) changes for values of x which are large as compared with the Debye radius a of the plasma without a beam. If we employ a computational method which is exactly the same as that used in (Ref. 6), we obtain the expression

$$E(x) = E_\infty\left\{1 + \frac{2}{\sqrt{3\pi}}\Omega_-^{-4/3}\left(\frac{x}{a}\right)^{1/3}\exp\left[-\frac{3}{4}\left(\frac{x}{\Omega_- a}\right)^{4/3}\right] \times \right. \qquad (3.7)$$
$$\left. \times \exp\left(i\left[\frac{3\sqrt{3}}{4}\left(\frac{x}{\Omega_- a}\right)^{4/3} + \frac{2\pi}{3}\right]\right)\right\} \quad \left(a = \left(\frac{\theta}{4\pi n e^2}\right)^{1/2}\right)$$

Expression (3.7) provides the law for the decrease in the difference $E(x) - E_\infty$, which is similar to that given in (Ref. 6). This would be expected, since there is no thermal velocity scatter in the beam. If $E_1 + 4\pi i f_1/\omega$ changes to zero, then the field E(x) strives to zero in the case of $x \to \infty$

according to an exponential law, which follows from a determination of the integrals by means of residues.

If we know the law by which the electric field is distributed in the plasma, we can find the distribution of the beam velocity perturbation and its density according to formulas (1.2) and (1.3). Let us transform these expressions into a form which is more advantageous for obtaining asymptotic formulas

$$v(x) = \frac{e}{mv_0} \exp \frac{i\omega x}{v_0} \int_x^{\infty} E(\xi) \exp\left(-\frac{i\omega\xi}{v_0}\right) d\xi$$

$$\rho(x) = -\frac{\rho_0}{v_0} v(x) + \frac{i\omega e\rho_0}{mv_0^3} \exp\left(\frac{i\omega x}{v_0}\right) \int_x^{\infty} E(\xi) \exp\left(-\frac{i\omega\xi}{v_0}\right)(\xi - x)\, d\xi \tag{3.8}$$

For example, in the case of large values of x the beam velocity perturba- /7 tion is related to the electric field E(x) by the simple relationship

$$v(x) = -\frac{ie}{m\omega} E(x) \tag{3.9}$$

4. Study of Resonance. Let us find the roots of the integrand denominators in (3.5). The dispersion equation $1-L_k-K_2(k)=0$ for longitudinal oscillations (Ref. 2) can be written in the following form

$$1 - \left(\frac{\beta\Omega_+}{\beta - \nu}\right)^2 = \beta^2\Omega_-^2 [J_-(\beta) - 1] \qquad \left(\nu = \frac{v_0}{u^*}\right) \tag{4.1}$$

Poles with small Im k make a significant contribution to the integral in (3.6). Therefore, we shall search for the roots of equation (4.1) which lie close to the essentially singular point k = 0 in the upper half-plane k. Assuming that $|\beta| \gg 1$, expanding $1/(\beta - \nu)$ in powers of ν/β, and employing the asymptotic form of the function $J_-(\beta)$ in the upper half-plane (Ref. 2) we finally obtain

$$\frac{1}{\beta} = \frac{-\Omega_+^2\nu \pm \sqrt{\Omega_+^4\nu^2 + 3(\Omega_-^2 + \Omega_+^2\nu^2)\varepsilon}}{3(\Omega_-^2 + \Omega_+^2\nu^2)} \tag{4.2}$$

The roots of equation $1 - L_{-k} - K_2(k) = 0$ are determined by the same formula (4.2), in which ν changes sign, however. One of the roots of (4.2) lies in the upper half-plane k only when the radicand is negative, i.e.,

$$\varepsilon < -\frac{\Omega_+^4 v^2}{3(\Omega_-^2 + \Omega_+^2 v^2)} \tag{4.3}$$

Let us find the roots of equation $1 - L_k - K_1(k) = 0$ which also lie in the upper half-plane k. This equation has the following form

$$1 - \left(\frac{\beta\Omega_+}{\beta - v}\right)^2 = \beta^2\Omega_-^2 [J_+(\beta) - 1] \tag{4.4}$$

The asymptotic form of $J_+(\beta)$ in the upper half-plane k (which corresponds to the lower half-plane β) has an exponentially small imaginary term (Ref. 2).

Let us represent the desired root of equation (4.4) in the following form

$$\beta = \beta_0 (1 + \beta_1 / \beta_0) \tag{4.5}$$

where β_0 is the real part of the root which determines (4.2), disregarding the exponentially small term, and β_1 is the small imaginary addition. We then have

$$\frac{\beta_1}{\beta_0} = -\frac{i\Omega_-^2 \sqrt{1/2\pi}\, \beta_0^5 \exp(-1/2\,\beta_0^2)}{3\Omega_-^2 + \Omega_+^2 v(3v + \beta_0)} \tag{4.6}$$

It follows from the formulation of the problem that $\Omega_+ \ll \Omega_-$ and $\nu \ll 1$. Let us represent the dielectric constant ε in the form $\varepsilon = \varepsilon_* + \Delta\varepsilon$, where

$$\varepsilon_* = -\frac{\Omega_+^4 v^2}{3(\Omega_-^2 + \Omega_+^2 v^2)} \tag{4.7}$$

As we shall see below, the value of ε_* will be critical, since in the case of $\varepsilon > \varepsilon_*$ the law by which the field changes close to the boundary differs qualitatively from that in the case of $\varepsilon < \varepsilon_*$. The critical value in (Ref. 6) was $\varepsilon_* = 0$. The shift in the critical value ε_* in the region of negative ε can be explained in the case under consideration by a frequency decrease for a moving beam due to the Doppler effect. Let us investigate the cases

$\Delta\varepsilon > 0$ and $\Delta\varepsilon < 0$. For $\Delta\varepsilon > 0$ we have

$$\frac{1}{\beta_0} = \frac{-\Omega_+^2\nu \pm \sqrt{3(\Omega_-^2 + \Omega_+^2\nu^2)\Delta\varepsilon}}{3(\Omega_-^2 + \Omega_+^2\nu^2)} \tag{4.8}$$

so that in the case of $\nu > 0$ and $\Delta\varepsilon > -\varepsilon_*$, $\beta_0 > 0$, and then we have the root /8 of the equation (4.4) with Im $\beta_1 < 0$. In the case of $\nu < 0$, there is always at least one such root of equation (4.4). Let $\Delta\varepsilon < 0$. Then in the lower half-plane of β there is always a root of equation (4.1)

$$\frac{1}{\beta} = \frac{-\Omega_+^2\nu + i\sqrt{3(\Omega_-^2 + \Omega_+^2\nu^2)|\Delta\varepsilon|}}{3(\Omega_-^2 + \Omega_+^2\nu^2)} \tag{4.9}$$

Taking the relationships (4.2)-(4.9) into consideration and confining ourselves to linear terms in expansion by powers of k, we find that in the case of $\Delta\varepsilon < 0$ the electric field close to the boundary changes according to the law

$$\begin{aligned} E = \frac{E_1}{\varepsilon}\Bigg\{1 - 2\,\frac{\sqrt{2/\pi}\,\Omega_-^2 + \Omega_+^2\nu}{\Omega_+^2\nu} \times \\ \times \exp\left[-\frac{x}{\Omega_-\varepsilon}\left(\frac{|\Delta\varepsilon|}{3(\Omega_-^2 + \nu^2\Omega_+^2)}\right)^{1/2}\right]\cos\left[\frac{x}{\Omega_-\varepsilon}\,\frac{\nu\Omega_+^2}{3(\Omega_-^2 + \nu^2\Omega_+^2)}\right]\Bigg\} \end{aligned} \tag{4.10}$$

In order to abbreviate formula (4.10), let us set $\nu_1 = 0$ and $f_1 = 0$.

We can obtain the behavior of the field close to the boundary in the case of $\Delta\varepsilon > 0$ in a similar way

$$E = \frac{E_1}{\varepsilon}\left\{1 + \frac{\varepsilon\gamma}{2\Omega_+^2\nu}\exp\left[\frac{ix}{\Omega_-\varepsilon\gamma} - \frac{\Omega_- x}{\Omega_+\varepsilon}\,\frac{\sqrt{\pi}}{\sqrt{2}}\,\frac{\gamma^{3/2}}{\sqrt{\nu\Delta\varepsilon}}\exp\left(-\frac{\gamma^2}{2}\right)\right]\right\}$$

$$\gamma = \frac{3(\Omega_-^2 + \Omega_+^2\nu^2)}{\Omega_+^2\nu} \tag{4.11}$$

Formula (4.11) was also obtained on the assumption that $\nu_1 = 0$ and $f_1 = 0$.

The necessity of taking into account the residue in the pole when determining the integral from zero to infinity, leading to expression (4.11), can be explained by the fact that the initial integration contour must be deformed when determining the asymptotic form of this integral by the method of descent, so that it coincides with the line of the level passing through saddle point. In the case of this deformation, the pole is bypassed in the right half of the upper half-plane, if it is located below the level line and above the abscissa axis.

It must be assumed that $\Delta\varepsilon$ is small in formula (4.11), but γ must therefore be large, so that the field is slowly damped with an increase in x. In the opposite case, this component can be disregarded, and the field is determined by (3.7).

In the absence of a beam, formulas (4.10) and (4.11), which describe the resonance case, are inapplicable. If quadratic terms are taken into account in the expansion in powers of k, then the passage to the limit to the case where there is no beam gives the relationships obtained in (Ref. 6).

In thecase of $x = 0$, formulas (4.10) and (4.11) do not provide the correct boundary value of E_1 due to the fact that terms on the order of $|\Delta\varepsilon|$ were disregarded in the computations. However, with an increase in x,in both cases the field undergoes oscillations around the value of E_1/ε, to which it strives at infinity.

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